UDC 656:02

MODEL AND METHOD FOR SOLVING THE TRANSPORT PROBLEM WITH A THRESHOLD CHANGE IN TRANSPORTATION TARIFFS

Slavych V. P., *Ph.D., Associate Professor, Associate Professor of the Department of Transport Systems and Technical Service of Kherson National Technical University, Kherson, Ukraine, e-mail: vslavich@ukr.net, ORCID: 0000-0001-7882-4198;*
Afonin M. O., *Ph.D., Associate Professor, Associate Professor of Transport Technologies Department in Lviv Polytechnic National University, Lviv, Ukraine, e-mail: maksym.o.afonin@lpnu.ua, ORCID: 0000-0001-5850-7478;*
Rusavska O. O., *Student of the Department of Transport Systems and Technical Service of Kherson National Technical University, Kherson, Ukraine, e-mail: s7998208@kntu.edu.ua.*

The article presents a transport problem with a threshold change in transportation tariffs, in which the cost of transportation depends on the volume of cargo between specific suppliers and consumers. The relevance of the study is due to the spread of flexible tariff mechanisms in modern transport and logistics systems, under which reduced or wholesale tariffs are applied when a certain volume of transportation is exceeded. Similar approaches are used in cargo consolidation systems, long-term contracts, multimodal transportation and in organizing regular deliveries of large batches of products. A mathematical model of the transport problem is proposed, in which for each "supplier-consumer" pair basic and reduced transportation tariffs are set, as well as threshold values of cargo volumes, after which the alternative delivery cost is applied. The objective function of the problem is constructed taking into account the dynamic change in transportation costs depending on the volume of transport flow and the system of balance constraints of the transport problem. A modified method for constructing a reference plan based on the classical least cost method has been developed, which takes into account the possibility of switching to reduced tariffs when reaching transportation threshold values. A modified potential method has also been proposed for finding the optimal plan, which provides for repeated updating of the effective cost matrix and recalculation of potentials after each cargo redistribution. Unlike the classical transport problem, in the proposed model the structure of transport costs can change directly in the process of forming a transportation plan. The practical significance of the study lies in the possibility of using the developed model to improve the efficiency of freight transportation management in the conditions of wholesale tariffs and dynamic changes in the cost of delivery.
Key words: *transport problem; transport logistics; potential method; reference plan; optimal plan; threshold tariffs; variable transportation cost; freight transportation.*

DOI: 10.33815/2313-4763.2026.1.32.164-173

Introduction. Effective management of transport flows is one of the key tasks of modern logistics systems, as it directly affects the level of costs, delivery speed and overall productivity of supply chains. The tasks of optimal distribution of goods between suppliers and consumers are traditionally formalized in the form of a linear programming transport problem, which allows minimizing the total transportation costs subject to balance constraints.

In the classical formulation of the transport problem, the cost of transporting a unit of cargo between a specific supplier and a consumer is considered a constant value that does not depend on the volume of the transport consignment. This approach is sufficient for basic planning models, but does not take into account important practical features of real transport and logistics systems.

In modern conditions of operation of transport networks, differentiated tariff policies are increasingly used, which provide for a decrease in the cost of transportation with an increase in the volume of cargo between individual participants in the logistics process. Such mechanisms are typical for wholesale transportation, long-term contracts, cargo consolidation systems, as well as for integrated logistics platforms, where the cost of delivery is a function of the intensity of the transport flow.

The presence of threshold values of transportation volumes, after which reduced tariffs are applied, leads to the fact that the structure of transport costs becomes dependent on the transportation plan itself. This, in turn, significantly complicates the mathematical model of the problem and makes it nonlinear in nature. Under such conditions, classical methods of constructing an optimal plan require significant modification, since the choice of transportation route affects not only the current cost, but also the future tariff structure in the system.

Thus, there is a need to develop new approaches to modeling transport problems that take into account the variability of transportation costs depending on the volume of transport flow. Of particular importance is the construction of algorithms that allow adapting classical optimization methods, in particular the potential method, to the conditions of threshold tariff functions without significantly complicating computational procedures.

In this regard, the study of transport problems with a variable transportation cost structure is a relevant scientific direction that has important applied significance for increasing the efficiency of management of modern transport and logistics systems.

Analysis of recent achievements and publications. In the work [1], a mathematical-information model of a special type of transport problem is proposed to increase the economic efficiency of freight transportation. The main attention is paid to the construction of a modified transport model and algorithms for determining rational transportation plans. However, the work does not consider the dependence of the cost of transportation on the size of the cargo and there is no mechanism for automatic change of tariffs when the specified threshold values are exceeded.

In [2], a modified transportation problem is considered in the case of grouping cargo suppliers and a method for finding reference and optimal transportation plans is proposed. The study is focused on the structural features of building a transport model, but it does not take into account the situation when the cost of transportation varies depending on the volume of transported cargo between specific suppliers and consumers.

In the study [3], a multi-criteria transportation problem is considered, taking into account product storage technologies and fuzzy sets. The work takes into account complex decision-making conditions and the multi-criteria nature of optimization, but it does not investigate the mechanism of the threshold change in transportation costs when certain transportation volumes are reached.

In the work [4], the problem of optimizing the process of cargo delivery of a gas supply enterprise was investigated. The main emphasis was placed on minimizing costs and improving the logistics processes of the enterprise. However, the work did not take into account the possibility of using different transportation tariffs depending on the size of the transported cargo.

In the article [5], a model of integrated design of a transport network and supply chain for perishable products using a fuzzy two-level decision support system is proposed. The work covers the issue of optimization of transport costs, but does not consider the transport problem with variable tariffs depending on threshold values of transport volumes.

In [6], a model of the transport problem in the case of cargo delivery by two different types of vehicles is considered. The study takes into account the features of the use of different types of transport and the corresponding tariffs, but the change in the cost of transportation depending on the quantity of cargo for a specific cell of the transport table is not considered.

The study [7] considers the problem of sustainable transport planning taking into account congestion and uncertain conditions. The main emphasis is on the adaptability of the transport system and taking into account external factors. However, the work does not propose mechanisms for changing the cost of transportation depending on the size of the transported cargo.

In [8], approaches to modeling transportation processes under variable system parameters and adaptive redistribution of freight flows are considered; however, the dependence of transportation tariffs on shipment volume and the mechanism of threshold-based cost changes for individual supplier–consumer pairs are not taken into account.

In the article [9] the problem of vehicle routing for combined passenger and freight transportation is considered. The main attention is paid to routing and coordination of transportation, however, there is no model of the change in the cost of delivery depending on the amount of cargo transported.

The paper [10] provides an overview of the use of artificial intelligence in logistics optimization problems. The study covers a wide range of modern methods for optimizing transport processes, but the issue of constructing a transport problem with variable threshold tariffs is not considered separately.

In the article [11], optimization of freight transport routes in dynamic networks with carbon emissions in mind is investigated. The authors take into account environmental criteria and variable traffic conditions, but the mechanism of dependence of the transport tariff on the size of the cargo is absent in the work.

In [12], a multi-criteria multi-period routing problem for the delivery of perishable goods is considered. The proposed approach takes into account the level of consumer satisfaction and delivery time parameters, but does not take into account the change in transportation costs depending on the volume of transportation.

In [13], a comparative analysis of routing optimization algorithms in pharmaceutical supply chains is conducted. The main emphasis is on the efficiency of routing algorithms, but the transport problem with variable tariffs when threshold values are exceeded is not investigated.

In [14], a hybrid intelligent transport system for optimizing transportation routes is proposed. The study focuses on the use of intelligent decision-making methods, but the issue of the threshold change in transport costs depending on the volume of transportation is not considered.

The article [15] provides an overview of vehicle routing problems and corresponding algorithms for logistics systems. The work systematizes modern approaches to transport optimization, but does not contain models of the transport problem with variable tariffs depending on the size of the cargo between suppliers and consumers.

A comparative analysis of the reviewed studies has shown that most existing transportation models are based on the assumption of fixed transportation tariffs or take into account additional factors in the form of fuzzy parameters, multicriteria optimization, fixed costs, or routing constraints. In contrast to these approaches, the proposed study considers a transportation problem in which the transportation cost directly depends on the volume of the transport flow and changes when predetermined threshold values are reached. Such an approach makes it possible to model wholesale tariff mechanisms and discount systems that are practically not considered in existing transportation models and are not taken into account in classical transportation problem formulations.

Thus, the analysis of modern scientific research has shown that, despite the significant development of transportation models and optimization methods, insufficient attention has been paid to transportation problems in which transportation costs depend on cargo volumes and change when predetermined threshold values are reached. In addition, existing studies do not propose approaches for constructing initial feasible and optimal transportation plans under conditions of dynamic tariff changes during cargo redistribution. This creates the need for developing appropriate mathematical models and solution methods adapted to threshold tariff mechanisms.

The scientific novelty lies in the development of a transportation problem with a variable structure of transportation tariffs, in which the effective transportation cost matrix is iteratively dependent on the current transportation plan and is updated during its construction. Additionally, a modified method for constructing an initial feasible solution is developed, and the potential method is improved, both involving dynamic recalculation of effective costs and potentials at each step of the solution process.

Goal and problem statement. In modern transport and logistics systems, flexible tariff mechanisms are increasingly used, in which the cost of transportation depends on the volume of the transport consignment. In the practice of freight transportation, situations are common when, when a certain volume of delivery is exceeded, reduced or preferential tariffs are applied for individual routes. Similar approaches are used in wholesale transportation systems, long-term contracts, cargo consolidation, multimodal transport schemes and in organizing regular deliveries of large volumes of products.

The classical transport problem involves the use of a constant matrix of transportation costs, in which the tariff for each “supplier-consumer” pair does not depend on the volume of the transport flow. This formulation is effective for problems with fixed transport tariffs, but it does not allow taking into account the change in the cost of transportation with an increase in the volume of delivery. As a result, the optimal plan obtained by classical methods may not correspond to the real economic conditions of the transport system, since it does not take into account the possibility of reducing costs through the use of wholesale tariffs.

In this regard, there is a need to develop a mathematical model of the transport problem, in which the cost of transportation is determined not only by the transportation route, but also by the amount of cargo transported. A feature of such a problem is that the structure of transport costs can change directly in the process of forming a transportation plan depending on the achievement of threshold values of transport flows.

The aim of the study is to develop a mathematical model of the transportation problem with a variable structure of transportation tariffs, in which the effective transportation cost matrix is iteratively dependent on the current transportation plan and is updated during its formation, as well as to develop methods for finding an initial feasible solution and an optimal plan based on modifications of the least cost method and the potential method.

To achieve the set goal, it is planned to:

– formalize the dependence of transport costs on the volume of transportation;

– construct the objective function of the problem taking into account the threshold change in tariffs;

– develop a modified method for constructing a reference transportation plan;

– to improve the potential method for problems with a variable structure of transport costs;

– to form an algorithm for iterative improvement of the transportation plan, taking into account the dynamic change in tariffs in the process of redistribution of cargo flows.

Thus, the problem statement consists in determining a transportation plan that satisfies the balance constraints of the transport system and ensures the minimization of total transportation costs under the conditions of applying threshold tariffs and changing the cost of transportation depending on the volume of transport flow.

The proposed model is intended for freight transportation planning problems in logistics systems with wholesale or contract-based tariffs, particularly for distribution of homogeneous goods between suppliers and consumers in centralized supply systems, logistics hubs, and distribution networks where discounted tariffs are applied when shipment volumes exceed predefined thresholds on individual routes.

Presentation of the main research material. Let there be given m cargo suppliers $A_1, A_2, \dots, A_i, \dots, A_m$, which have cargo stocks concentrated in the quantities $a_1, a_2, \dots, a_m$ of units, respectively. This cargo must be transported to n consumers $B_1, B_2, \dots, B_j, \dots, B_n$, whose needs are in $b_1, b_2, \dots, b_n$ the units, respectively.

Unlike the classical transportation problem, in the proposed model the cost of transportation depends not only on the transportation route, but also on the volume of cargo transported between the corresponding supplier and consumer.

For each supplier-consumer pair, the following values are set:

1) the basic cost of transporting a unit of cargo;

2) reduced (preferential) transportation cost;

3) the threshold value of the transportation volume, after which a reduced tariff is applied.

The basic transportation costs are given by the matrix:

$$C = \begin{pmatrix} C_{11} & C_{12} & \dots & C_{1n} \\ C_{21} & C_{22} & \dots & C_{2n} \\ \dots & \dots & \dots & \dots \\ C_{m1} & C_{m2} & \dots & C_{mn} \end{pmatrix}, \qquad (1)$$

where c_{ij} – the cost of transporting a unit of cargo from the cargo supplier A_i to the consumer of the cargo B_j ($i = \overline{1,m}$; $j = \overline{1,n}$).

Reduced transportation costs are given by the matrix:

$$C' = \begin{pmatrix} c'_{11} & c'_{12} & \dots & c'_{1n} \\ c'_{21} & c'_{22} & \dots & c'_{2n} \\ \dots & \dots & \dots & \dots \\ c'_{m1} & c'_{m2} & \dots & c'_{mn} \end{pmatrix}. \quad (2)$$

Moreover, the following condition is met:

$$\forall_{i,j}\, c'_{ij} < c_{ij}.$$

The threshold values of transportation volumes are given by the matrix:

$$H = \begin{pmatrix} h_{11} & h_{12} & \dots & h_{1n} \\ h_{21} & h_{22} & \dots & h_{2n} \\ \dots & \dots & \dots & \dots \\ h_{m1} & h_{m2} & \dots & h_{mn} \end{pmatrix} \quad (3)$$

where h_{ij} – the minimum transportation volume, after which $(i;j)$ a reduced tariff is applied for the route.

Let us denote the transportation plan by:

$$X = \{x_{11}; x_{12}; \dots; x_{1n}; x_{21}; x_{22}; \dots; x_{2n}; \dots; x_{m1}; x_{m2}; \dots; x_{mn}\}, \quad (4)$$

where x_{ij} – the amount of cargo transported from supplier A_i to consumer B_j.

Then the actual cost of transporting a unit of cargo for the route $(i;j)$is determined by the relationship:

$$\tilde{c}_{ij}(x_{ij}) = \begin{cases} c_{ij}, & x_{ij} \le h_{ij}, \\ c'_{ij}, & x_{ij} > h_{ij}. \end{cases} \quad (5)$$

In this case, when the threshold value h_{ij} is exceeded, the reduced tariff c'_{ij} is applied to the entire transportation volume x_{ij} for the corresponding supplier–consumer pair, i.e., the tariff switch has a full (non-marginal) effect.

Thus, when the established threshold value is exceeded, an alternative (reduced) transportation tariff is used for the corresponding route.

It is necessary to find a transportation plan (4) that:

1) ensures full removal of cargo from all suppliers;
2) satisfies the needs of all consumers;
3) minimizes the total cost of transportation taking into account the threshold change in tariffs.

The objective function of the problem has the form:

$$F = \tilde{c}_{11}(x_{11}) \cdot x_{11} + \tilde{c}_{12}(x_{12}) \cdot x_{12} + \dots + \tilde{c}_{1n}(x_{1n}) \cdot x_{1n} + \tilde{c}_{21}(x_{21}) \cdot x_{21} + \tilde{c}_{22}(x_{22}) \cdot x_{22} +$$

$$+ \dots + \tilde{c}_{2n}(x_{2n}) \cdot x_{2n} + \dots + \tilde{c}_{m1}(x_{m1}) \cdot x_{m1} + \tilde{c}_{m2}(x_{m2}) \cdot x_{m2} + \dots + \tilde{c}_{mn}(x_{mn}) \cdot x_{mn} =$$

$$= \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij}(x_{ij}) \cdot x_{ij} \to min \quad (6)$$

Taking into account expression (5), the objective function (6) can be written in expanded form:

$$F = \sum_{i=1}^{m} \sum_{j=1}^{n} \begin{cases} c_{ij} x_{ij}, & x_{ij} \le h_{ij} \\ c'_{ij} x_{ij}, & x_{ij} > h_{ij} \end{cases} \to min \quad (7)$$

It should be noted that due to the dependence of transportation costs on the volume of the transport flow x_{ij}, the proposed model belongs to the class of piecewise-linear transportation problems. The objective function contains threshold points of tariff changes, at which a transition from the basic transportation cost to a reduced (discounted) cost occurs. Thus, unlike the classical linear transportation problem, the proposed model is characterized by a piecewise-linear cost function whose parameters change depending on transportation volumes.

Optimization is performed under the balance constraints of the transport problem:

$$\begin{cases} \sum_{j=1}^{n} x_{ij} = a_i,\ i = \overline{1,m}, \\ \sum_{i=1}^{m} x_{ij} = b_j,\ j = \overline{1,n}, \\ x_{ij} \geq 0. \end{cases} \quad (8)$$

Thus, the proposed mathematical model of the transport problem takes into account the change in transportation tariffs depending on the volume of transport flow between individual suppliers and consumers. Unlike the classical transport problem, in which the cost matrix is constant, in the proposed model the structure of transport costs changes in the process of forming a transport plan depending on the achievement of threshold values of supply volumes. This allows formalizing transport planning problems in the conditions of applying wholesale tariffs, discount systems or contractual terms of transport.

Based on the formulated mathematical model of the transport problem with threshold values of the transportation cost, we will consider the procedure for constructing a reference plan, which is an adaptation of the classical least cost method to the conditions of variable (threshold) tariffs.

Method and algorithm for finding the reference plan of a modified transportation problem.

To build the reference plan, a modified least cost method is used, which takes into account the change in transportation costs depending on the volume of cargo between a specific supplier-consumer pair.

In each cell of the transport table, the possible volume of transportation is defined as:

$$x_{ij}^{\max} = \min(A_i, B_j). \quad (9)$$

The rule for determining the cost of transportation. We enter the following parameter $\tilde{c}_{ij}$ - the effective cost of transportation, which will be determined under the condition of the following threshold transition:

$$\tilde{c}_{ij} = \begin{cases} c_{ij}, & x_{ij}^{\max} \leq h_{ij}, \\ c'_{ij}, & x_{ij}^{\max} > h_{ij}. \end{cases} \quad (10)$$

Thus, the transportation cost for each cell is dynamic and depends on the possible load volume.

Algorithm for constructing a reference plan.

Step 1. Initial analysis of the transport table. For all free cells, the possible volume of transportation is determined $x_{ij}^{\max}$and the threshold condition is checked h_{ij}. Based on this, the current matrix of effective costs is formed $\tilde{c}_{ij}$.

Step 2. Selecting the minimum value. Among all free cells $(i_0; j_0)$, a cell is selected for which:

$$\tilde{c}_{i_0 j_0} = \min \tilde{c}_{ij}.$$

Step 3. Destination of transportation. The volume is recorded in the selected cell:

$$x_{i_0 j_0} = \min(A_{i_0}, B_{j_0}).$$

After that, an adjustment is made to the inventory and demand balances, and the corresponding row or column is excluded from further consideration (in case of complete exhaustion).

Step 4. Update values. After each step, all remaining cells are rechecked:

1) the possible volume of transportation is calculated;

2) the fulfillment of the threshold condition is checked;

3) the effective cost is updated $\tilde{c}_{ij}$.

Step 5. Iterative process. Steps 2–4 are repeated until all consumer needs are fully satisfied and suppliers' stocks are exhausted.

The proposed algorithm differs from the classical least cost method in that the choice is based not on static cost values, but on a dynamically changing matrix of effective costs, which takes into account the possible transition to a reduced (wholesale) price when a threshold transportation volume is reached.

The next stage after building a reference plan and taking into account threshold changes in transportation costs is the procedure for improving it in order to achieve optimal distribution of cargo flows.

Method for finding the optimal plan (modified potential method).

After constructing the reference plan, its improvement is performed based on the modified potential method, adapted to the conditions of variable (threshold) transportation costs.

Step 1. Checking the reference plan for degeneracy . The following condition is checked for the constructed reference plan:

$$N = m + n - 1,$$

where N – the number of occupied cells.

In case of violation of this condition, zero (fictitious) transportations are entered into the corresponding cells without violating the structure of the plan.

Step 2. Calculation of potentials. A system of potential equations is constructed for all occupied cells:

$$u_i + v_j = \tilde{c}_{ij},$$

where $\tilde{c}_{ij}$is the effective cost of transportation, which is determined taking into account the threshold rule.

The potentials u_i, $v_j$are determined in the classical way with one of them fixed (for example, $u_1 = 0$).

Step 3. Checking the optimality conditions. For all free cells, the score is calculated:

$$\Delta_{ij} = \tilde{c}_{ij} - (u_i + v_j).$$

– If $\Delta_{ij} \geq 0$for all free cells, the plan is optimal.

– If there are cells with $\Delta_{ij} < 0$– the plan needs to be improved.

Step 4. Selecting a cell to improve the plan. In case of violation of optimality, a set of cells is considered for which:

$$\Delta_{ij} < 0.$$

For each such cell, a closed load redistribution cycle is built.

A feature of the modified approach is that the assessment $\Delta_{ij}$is determined taking into account the current effective cost $\tilde{c}_{ij}$, which may change in the process of cargo redistribution.

Step 5. Construction of a cycle and redistribution of the load. For the selected cell, a cycle is constructed in which the vertices are alternately marked with the signs "+" and "–".

It is further defined:

$$\theta = \min(x_{ij} \text{ in cells with a "–"}),$$

after which redistribution is performed:

– in cells with a "+" sign: $x'_{ij} = x_{ij} + \theta$;

– in cells with a "–" sign: $x'_{ij} = x_{ij} - \theta$.

Step 6. Update transportation costs. After each redistribution, a threshold condition check is performed:

– if after increasing or decreasing the volume the condition is met $x'_{ij} > h_{ij}$, then the reduced value is applied c'_{ij};

– if the condition is not met, the base value is used c_{ij}.

As a result, the effective value matrix changes $\tilde{c}_{ij}$, which requires recalculation of the potentials.

Step 7. Repeated iteration. After updating the plan, the algorithm returns to the stage of calculating potentials and checking optimality.

The procedure is repeated until the condition is met for all free cells:

$$\Delta_{ij} \geq 0.$$

Thus, the proposed method is a modification of the classical method of potentials for the case when the cost of transportation depends on the volume of cargo and can change when the specified threshold values are reached. Unlike the classical approach, during each iteration, dynamic updating of individual elements of the cost matrix is possible due to the transition between regular and wholesale tariffs. This, in turn, leads to the need to recalculate the potentials and re-check the optimality conditions after each redistribution of cargo in the cycle. This approach allows you to take into account the impact of variable tariffs on the total cost of transportation and provides the opportunity to find a more cost-effective transportation plan under threshold pricing conditions.

Conclusions. Thus, the paper considers a modified transport problem in which the cost of transportation depends on the volume of cargo between specific suppliers and consumers. The proposed model takes into account the threshold effect of tariff changes, which allows formalizing situations of wholesale transportation using reduced (discounted) costs.

It has been shown that the introduction of threshold tariff values leads to the formation of a piecewise-linear transportation problem model in which the structure of the cost function changes depending on transportation volumes. This makes it possible to describe more adequately the real operating conditions of transportation and logistics systems that apply wholesale and preferential tariff schemes.

A modified method for constructing a reference plan has been developed, which is based on the classical least cost approach, but supplemented by a procedure for preliminary verification of the possible volume of transportation and appropriate cost adjustment. This ensures a more correct choice of transport routes at the initial stage of solving the problem.

A modified potential method is also proposed for finding the optimal plan, which takes into account the dynamic change of the cost matrix in the process of iterative redistribution of loads. It is shown that this feature leads to the need to recalculate the potentials and check the optimality conditions after each cycle.

The proposed model allows taking into account threshold values of transportation volume and correctly describing the economic conditions of transport processes, expanding the capabilities of classical transport optimization methods.

The developed model can be applied in freight transportation practice to simulate real logistics systems with wholesale discounts and restrictions on transportation volumes.

Prospects for further research. Further research should be directed at expanding the proposed model by introducing stochastic or fuzzy parameters of transportation costs and threshold values of cargo volumes. This will allow taking into account the uncertainty of real logistics processes and increasing the adequacy of the model in practical application.

A separate direction of further research is the software implementation of the proposed algorithm and conducting numerical experiments to analyze the behavior of the model for different structures of transport problems and different values of threshold levels. This will allow assessing the stability of the algorithm and the features of the formation of an optimal plan in conditions of dynamic changes in transportation costs.

REFERENCES

1. Slavych, V. P., Dobrova, K. D. (2020). Modelʹ ta metod znakhodzhennya opornoho ta optimalʹnykh planiv modifikovanoyi transportnoyi zadachi u vypadku hrupuvannya postachalʹnykiv vantazhu. *Prykladni pytannya matematychnogo modelyuvannya*. No 1. P. 187–193. http://doi.org/10.32782/2618-0340/2020.1-3.19.

2. Slavych, V. P., Biloushchenko, D. Yu. (2025). Model of the transport problem in the case of cargo delivery by two different types of vehicles. *Naukovyi visnyk KhDMA*. No 31. P. 178–186. http://doi.org/10.33815/2313-4763.2025.2.31.178-186.

3. Bera, R. K., Mondal, S. K. (2022). A multi-objective transportation problem with cost dependent credit period policy under Gaussian fuzzy environment. *Operational Research*. 22(4): 3147–3182. http://doi.org/10.1007/s12351-022-00691-4.

4. Bera, R. K., Mondal, S. K. (2023). A multi-objective transportation problem under quantity dependent credit period and cost structure policies in triangular intuitionistic fuzzy environment. *Engineering Applications of Artificial Intelligence*. 123: 106396. http://doi.org/10.1016/j.engappai.2023.106396.

5. Gütmen, S., Roy, S. K., Weber, G. W. (2024). An overview of weighted goal programming: a multi-objective transportation problem with some fresh viewpoints. *Central European Journal of Operations Research.* 32(3): 557–568. http://doi.org/10.1007/s10100-023-00861-5.

6. Luo, R., Ji, S., Ji, T. (2021). An effective chaos-driven differential evolution for multi-objective unbalanced transportation problem considering fuel consumption. *Applied Soft Computing*. 101: 107058. http://doi.org/10.1016/j.asoc.2020.107058.

7. Slavych, V. P. et al. (2025). Mathematical and information model of a special type of transport task as a tool for increasing economic efficiency of freight transportation. *Artificial intelligence as a tool to protect the economy from disinformation: innovative solutions and international practices*. Riga, Latvia: Baltija Publishing. 258 p. http://doi.org/10.30525/978-9934-26-586-0.

8. Mondal, M., Mandal, G., Shaikh, A. A., & Bhunia, A. K. (2025). Investigation of a multi-objective fixed charge transportation problem with quantity dependent transportation cost and discount policy *via* metaheuristics. *INFOR: Information Systems and Operational Research*, *63*(3), 562–591. https://doi.org/10.1080/03155986.2025.2492739.

9. Ghosh, S., Küfer, K. H., Roy, S. K., Weber, G. W. (2023). Type-2 zigzag uncertain multi-objective fixed-charge solid transportation problem: time window vs. preservation technology. *Central European Journal of Operations Research*. 31(1): 337–362. http://doi.org/10.1007/s10100-022-00811-7.

10. Ghosh, S., Roy, S. K. (2021). Fuzzy-rough multi-objective product blending fixed-charge transportation problem with truck load constraints through transfer station. *RAIRO-Oper Res*. 55: S2923–S2952. http://doi.org/10.1051/ro/2020129.

11. Ghosh, S., Roy, S. K., Verdegay, J. L. (2022). Fixed-charge solid transportation problem with budget constraints based on carbon emission in neutrosophic environment. *Soft Computing*. 26(21): 11611–11625. http://doi.org/10.1007/s00500-022-07442-9.

12. Haque, S., Bhurjee, A. K., Kumar, P. (2022). Multi-objective non-linear solid transportation problem with fixed charge, budget constraints under uncertain environments. *Systems Science & Control Engineering*. 10(1): 899–909. http://doi.org/10.1080/21642583.2022.2137707.

13. Keshavarz E., Mahmoodirad A., Niroomand S. (2023). A transportation problem considering fixed charge and fuzzy shipping costs. *Decision Making Advances* . 1(1): 115–122. http://doi.org/10.31181/dma11202313.

14. Mahmoodirad, A., Niroomand, S. (2023). A heuristic approach for fuzzy fixed charge transportation problem. *Journal of Decision Analytics and Intelligent Computing*. 3(1): 139–147. http://doi.org/10.31181/jdaic10005092023m.

15. Mondal, A., Roy, S. K., Midya, S. (2023). Intuitionistic fuzzy sustainable multi-objective multi-item multi-choice step fixed-charge solid transportation problem. *Journal of Ambient Intelligence and Humanized Computing*. 14(6): 6975–6999. http://doi.org/10.1007/s12652-021-03554-6.

Славич В. П., Афонін М. О., Русавська О. О. МОДЕЛЬ І МЕТОД РОЗВ'ЯЗАННЯ ТРАНСПОРТНОЇ ЗАДАЧІ З ПОРОГОВОЮ ЗМІНОЮ ТАРИФІВ ПЕРЕВЕЗЕННЯ

У статті представлено транспортну задачу з пороговою зміною тарифів перевезення, у якій вартість транспортування залежить від обсягу вантажу між конкретними постачальниками та споживачами. Актуальність дослідження обумовлена поширенням у сучасних транспортно-логістичних системах гнучких тарифних механізмів, за яких при перевищенні певного обсягу перевезення застосовуються знижені або оптові тарифи. Подібні підходи використовуються у системах консолідації вантажів, довгострокових контрактах, мультимодальних перевезеннях та при організації регулярних поставок великих партій продукції. Запропоновано математичну модель транспортної задачі, у якій для кожної пари «постачальник–споживач» задаються базові та знижені тарифи перевезення, а також порогові значення обсягів вантажу, після перевищення яких застосовується альтернативна вартість доставки. Побудовано цільову функцію задачі з урахуванням динамічної зміни вартості перевезення залежно від величини транспортного потоку та системи балансових обмежень транспортної задачі. Розроблено модифікований метод побудови опорного плану на основі класичного методу найменшої вартості, який враховує можливість переходу до знижених тарифів при досягненні порогових значень перевезення. Запропоновано також модифікований метод потенціалів для знаходження оптимального плану, у якому передбачено повторне оновлення матриці ефективних вартостей і перерахунок потенціалів після кожного перерозподілу вантажу. На відміну від класичної транспортної задачі, у запропонованій моделі структура транспортних витрат може змінюватися безпосередньо в процесі формування плану перевезень. Практичне значення дослідження полягає у можливості використання розробленої моделі для підвищення ефективності управління вантажними перевезеннями в умовах застосування оптових тарифів і динамічної зміни вартості доставки.

Ключові слова: *транспортна задача; транспортна логістика; метод потенціалів; опорний план; оптимальний план; порогові тарифи; змінна вартість перевезення; вантажні перевезення.*

Статтю прийнято до редакції 11.05.2026