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MODEL OF THE TRANSPORT PROBLEM IN THE CASE OF CARGO DELIVERY BY TWO DIFFERENT TYPES OF VEHICLES

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The article considers the features of modeling the transport problem in the case when the cargo can be delivered by two different types of vehicles. The classical transport model does not take into account the variability of transportation costs by type of transport, which requires the construction of special mathematical models and methods for reducing them to a standard form. The aim of the work is to construct special types of models of the transport problem in the case of cargo delivery by the number of types of vehicles, more than two, and to propose algorithms for solving such transport problems by reducing them using appropriate procedures to the classical model. Three approaches to the formalization of multimodal transportation have been developed. The first approach is based on the sequential transportation of cargo by both types of transport, which involves the formation of a final cost matrix as the sum of the corresponding elements of a three-dimensional matrix. The second approach involves the choice of the type of transport that provides the lowest delivery cost for each individual "supplier–consumer" connection. The third approach is based on the introduction of a probability matrix or partial cargo shares, which determine what part of the cargo should be transported by each mode of transport. For each of the models, procedures are proposed for reducing the three-dimensional cost structure to a two-dimensional form, which allows the application of the potential method to find the optimal transportation plan. The numerical examples presented confirm the correctness and practical applicability of the proposed models. The results obtained can be used to optimize logistics schemes, manage multimodal transportation, and also to develop software tools to support decision-making in transport systems.

Key words*: transport problem; interaction of transport modes; potential method; multimodal transportation; freight transportation; optimal delivery plan.*

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Introduction. The transportation problem is one of the basic tools of optimization analysis in logistics and operations research. Its classical formulation involves the delivery of cargo from suppliers to consumers by a single mode of transport, provided that the volumes and costs of transportation are known. However, the real practice of transportation is much more complicated: in many cases, cargo is transported by several types of vehicles, which significantly affects the cost structure and organization of the logistics process. Combined, multimodal or intermodal transportation allows to increase the flexibility of transportation schemes, but at the same time imposes additional requirements on mathematical models, which must take into account different transportation options, their cost, availability and limitations.

Scientific sources mainly consider modifications of the transport problem that take into account time, resource or technological parameters, however, the problem of formalizing the delivery of goods by several modes of transport is still insufficiently studied.

In particular, an approach to building a model in which the cost of transportation between each "supplier-consumer" pair is presented in the form of a multidimensional structure is required, and the optimization process requires reducing such a problem to a form suitable for the application of classical solution methods.

In this context, it is relevant to develop models of the transport problem for cases where the cargo can be transported by two or more types of vehicles, as well as to determine algorithms for their effective solution. This work is devoted to these issues, in which several approaches to building a model are proposed and methods for transforming multidimensional data into a classical transport formulation are demonstrated.

Analysis of recent achievements and publications. Recent studies indicate an increased focus on the development of multimodal and combined transportation, which form the basis for improving mathematical models of transport processes. The works [1–4] focus on conceptual approaches to the organization of multimodal chains, determining the criteria for their effectiveness and the impact of digitalization and integration with European logistics practices on transport systems. In particular, the review [4] summarizes modern optimization methods and demonstrates the trend of transition from general logistics concepts to formalized models using mathematical and algorithmic tools, while [3] emphasizes the role of digital technologies in improving the efficiency of multimodal freight transportation.

Important for the research topic are works directly related to the modification of the transport problem. Publications [5], [6] consider adaptations of the classical transport model for specific conditions, in particular grouping of suppliers and optimization of delivery at individual enterprises. These approaches form a methodological basis for building models with the reduction of multidimensional cost structures to two-dimensional ones, which is important in the context of cargo delivery by various modes of transport.

Modern works on synchromodal and combined transportation emphasize the need for flexible choice of transport mode depending on resource availability, network conditions, and cost. Publications [7–9] demonstrate the possibilities of combining optimization models with simulation, modular routing concepts, and heuristic algorithms to achieve lower logistics costs and better utilization of transport resources. Works [10], [11] also consider dynamic route updating and the application of machine learning methods, in particular Q-learning, which allows increasing the efficiency of transport systems in changing conditions.

Researchers pay considerable attention to uncertainty in logistics processes, equilibrium conditions, and environmental requirements. Publications [11–16] emphasize the need to build robust and low-carbon models of transport routes under uncertain demand and time-varying conditions. In particular, [12], [14] investigate equilibrium and optimization models for multimodal networks with intermodality and uncertainty, while industry-oriented examples [13], [15] consider cold chain logistics and perishable goods, demonstrating the practical importance of taking into account technical, environmental, and technological constraints.

The analysis of sources shows that despite the significant development of multimodal and synchromodal approaches, specialized formal models that describe the case of cargo delivery by two different modes of transport and the procedure for reducing the three-dimensional cost matrix to the classical transport problem have not been studied sufficiently. The works closest to this topic are [5], [6], however, they consider other aspects of transport model modification. Thus, the topic of the article is relevant and fills an important gap in modern research, creating prerequisites for the further development of models and optimization algorithms in the field of multimodal transport.

Goal and problem statement. In the practice of freight transportation, quite often the cargo has to be delivered not by one, but by two or more different types of transport, for example, by road and rail. In this case, it is obvious that the cost of transporting cargo from suppliers to consumers for different types of transport will be different. Therefore, when applying the model of the transport problem and the method of its solution for such situations, the necessary changes are required.

The purpose of this work is to construct special types of models of the transport problem in the case of cargo delivery by the number of types of vehicles, more than two, and to propose algorithms for solving such transport problems by reducing them to the classical model using appropriate procedures.

Presentation of the main research material. Let there be given *m* cargo suppliers $A_1, A_2, \ldots, A_i, \ldots, A_m$, in which cargo is concentrated in quantities $a_1, a_2, \ldots, a_m$ units, respectively. This cargo must be transported to *n* cargo consumers $B_1, B_2, \ldots, B_j, \ldots, B_n$, *who need it in quantities* $b_1, b_2, \ldots, b_n$ *units*, respectively. The cargo can be transported by two types of vehicles and the cost of transporting a unit of cargo by each type is known, which are given in the form of a three-dimensional matrix:

$$\begin{pmatrix} C_{11}^1 & C_{12}^1 & \dots & C_{1n}^1 \\ C_{21}^1 & C_{22}^1 & \dots & C_{2n}^1 \\ \dots & \dots & \dots & \dots \\ C_{m1}^1 & C_{m2}^1 & \dots & C_{mn}^1 \\ \hline C_{11}^2 & C_{12}^2 & \dots & C_{1n}^2 \\ C_{21}^2 & C_{22}^2 & \dots & C_{2n}^2 \\ \dots & \dots & \dots & \dots \\ C_{m1}^2 & C_{m2}^2 & \dots & C_{mn}^2 \end{pmatrix}, \quad (1)$$

or abbreviated:

$$C_{ij}^k,$$

where i – the number of the cargo supplier, $i = \overline{1,m}$;
j – the number of the cargo consumer, $j = \overline{1,n}$;
k – type of vehicle, $k = \overline{1,2}$.

It is necessary to draw up a transportation plan that allows you to transport all cargo from suppliers, satisfy the needs of all consumers, and has the lowest cost.

Let us denote the transportation plan by:

$$X = \begin{Bmatrix} x_{11}^1, x_{12}^1, \dots, x_{1n}^1, x_{21}^1, x_{22}^1, \dots, x_{2n}^1, \dots, x_{m1}^1, x_{m2}^1, \dots, x_{mn}^1, \\ x_{11}^2, x_{12}^2, \dots, x_{1n}^2, x_{21}^2, x_{22}^2, \dots, x_{2n}^2, \dots, x_{m1}^2, x_{m2}^2, \dots, x_{mn}^2, \end{Bmatrix}, \quad (2)$$

where x_{ij}^k – the amount of cargo transported from supplier i to consumer j by mode of transport k.

Let's build a model of this problem.

The objective function of such a problem will be:

$$\begin{gathered} F = C_{11}^1 \cdot x_{11}^1 + C_{12}^1 \cdot x_{12}^1 + \dots + C_{1n}^1 \cdot x_{1n}^1 + C_{21}^1 \cdot x_{21}^1 + C_{22}^1 \cdot x_{22}^1 + \dots + C_{2n}^1 \cdot x_{2n}^1 + \dots + \\ + C_{m1}^1 \cdot x_{m1}^1 + C_{m2}^1 \cdot x_{m2}^1 + \dots + C_{mn}^1 \cdot x_{mn}^1 + C_{11}^2 \cdot x_{11}^2 + C_{12}^2 \cdot x_{12}^2 + \dots + C_{1n}^2 \cdot x_{1n}^2 + \\ + C_{21}^2 \cdot x_{21}^2 + C_{22}^2 \cdot x_{22}^2 + \dots + C_{2n}^2 \cdot x_{2n}^2 + \dots + C_{m1}^2 \cdot x_{m1}^2 + C_{m2}^2 \cdot x_{m2}^2 + \dots + C_{mn}^2 \cdot x_{mn}^2 = \\ = \sum_{k=1}^{2} \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij}^k \cdot x_{ij}^k \to \min. \end{gathered}$$

Let us consider several different submodels of such a problem, depending on the specific conditions under which the cargo can be redistributed across different modes of transport.

1. First, let's consider the simplest option of using two types of transport.

Let's assume that cargo is delivered from each supplier to each consumer first by one mode of transport, then by another.

It is obvious that in such a situation, the amount of cargo transported between a specific supplier and consumer by the first and second modes of transport will be the same, because the same cargo is transported, that is:

$$C_{ij}^1 = C_{ij}^2,$$

where i and j are fixed.

In this case, to reduce such a problem to the classical interpretation, it is necessary to create a two-dimensional matrix of transportation costs, each element of which is the sum of two corresponding elements from the initial three-dimensional matrix of transportation costs (1), which are located in the same line relative to the coordinate axis of the mode of transport.

The new cost matrix will look like this:

$$\begin{pmatrix} C_{11}^1 + C_{11}^2 & C_{12}^1 + C_{12}^2 & \dots & C_{1n}^1 + C_{1n}^2 \\ C_{21}^1 + C_{21}^2 & C_{22}^1 + C_{22}^2 & \dots & C_{2n}^1 + C_{2n}^2 \\ \dots & \dots & \dots & \dots \\ C_{m1}^1 + C_{m1}^2 & C_{m2}^1 + C_{m2}^2 & \dots & C_{mn}^1 + C_{mn}^2 \end{pmatrix}. \quad (3)$$

Further, to find the optimal cargo transportation plan, you can apply known methods for solving the transport problem, for example, the most common potential method.

Consider numerical example 1.

Find the cheapest transportation plan for the transportation problem under the following conditions. Let there be 4 cargo suppliers A_1, A_2, A_3, A_4, which have cargo concentrated in quantities of 200, 250, 150, 300 units, respectively.

This cargo needs to be transported to 5 cargo consumers B_1, B_2, B_3, B_4, B_5, who need it in quantities of 200, 200, 100, 250, 150 units, respectively. The cargo can be transported by two types of vehicles and the cost of transporting a unit of cargo by each type of transport is given in the form of a three-dimensional matrix:

$$\begin{pmatrix} 5 & 6 & 4 & 7 & 8 \\ 7 & 3 & 4 & 9 & 5 \\ 5 & 8 & 7 & 6 & 4 \\ 6 & 4 & 6 & 7 & 9 \\ \hline 8 & 9 & 4 & 5 & 6 \\ 6 & 7 & 8 & 3 & 4 \\ 3 & 7 & 4 & 5 & 5 \\ 4 & 9 & 8 & 7 & 4 \end{pmatrix}.$$

To solve such a problem, we first construct the final cost matrix according to expression (3):

$$\begin{pmatrix} 13 & 15 & 8 & 12 & 14 \\ 13 & 10 & 12 & 12 & 9 \\ 8 & 5 & 11 & 11 & 9 \\ 10 & 13 & 14 & 14 & 13 \end{pmatrix}. \quad (4)$$

Next, we compile a planning table for the potential method, using the values of matrix (4) as transportation costs. The algorithm of the potential method itself is applied, the results of which are given in Table 1.

Table 1 – Planning table for the potential method of the transport problem for example 1

Planning matrix	***Consumers***					***Stocks***
Suppliers	B_1	B_2	B_3	B_4	B_5	
A_1	13	15	8 100	12 100	14	200
A_2	13	10 50	12	12 50	9 150	250
A_3	8	5 150	11	11	9	150
A_4	10 200	13	14	14 100	13	300
Needs	200	200	100	250	150	900

Thus, the optimal plan for the task was:

$$X = \{x_{13} = 100, x_{14} = 100, x_{21} = 50, x_{24} = 50, x_{34} = 150, x_{32} = 150, x_{41} = 200, x_{44} = 100,\}.$$

The value of the objective function is:

$$F = 100 \cdot 8 + 100 \cdot 12 + 50 \cdot 10 + 50 \cdot 12 + 150 \cdot 9 +$$
$$+ +150 \cdot 5 + 200 \cdot 10 + 100 \cdot 14 = 8600 .$$

2. Now let's consider another option for using two types of transport when using the transport problem.

Let's assume that the cargo can be transported by only one mode of transport. In this case, to select the mode of transport by which the cargo will be transported, the one with the cheapest transportation cost is chosen.

In this case, to reduce such a problem to the classical interpretation, it is necessary to create a two-dimensional matrix of transportation costs, each element of which is a minimum of two corresponding elements from the initial three-dimensional matrix of transportation costs (1), which are located in the same line relative to the coordinate axis of the mode of transport.

The new cost matrix will look like this:

$$\begin{pmatrix} min(C_{11}^1; C_{11}^2) & min(C_{12}^1; C_{12}^2) & \dots & min(C_{1n}^1; C_{1n}^2) \\ min(C_{21}^1; C_{21}^2) & min(C_{22}^1; C_{22}^2) & \dots & min(C_{2n}^1; C_{2n}^2) \\ \dots & \dots & \dots & \dots \\ min(C_{m1}^1; C_{m1}^2) & min(C_{m2}^1; C_{m2}^2) & \dots & min(C_{mn}^1; C_{mn}^2) \end{pmatrix}. \quad (5)$$

Further, to find the optimal cargo transportation plan, the potential method of solving the transport problem can be applied.

When solving a specific numerical problem, when writing the final matrix (5) and then filling in the planning table of the potential method, in the values of transportation costs, it is necessary to indicate which type of transport each of them applies to. This can be written with a superscript indicating the number of the type of transport, for example, 6^{1} - that is, the cost of transportation is 6 units and the 1st type of transport will be used. Also, a similar notation should be indicated in the values of the transportation plan itself. This is necessary when forming a transportation plan for distributing cargo by type of transport.

It should be noted that in the case where the minimum for calculating a certain cost coincides for both modes of transport, then any of them can be chosen to draw up a transportation plan, unless the priority of a specific mode of transport is stipulated in advance.

Consider numerical example 2.

As input values for the transport problem, we will take the values from the previous example 1.

To solve this problem, we compose a final matrix of values according to expression (5):

$$\begin{pmatrix} 5^1 & 6^1 & 4^1 & 5^2 & 6^2 \\ 6^2 & 3^1 & 4^1 & 3^2 & 4^2 \\ 3^2 & 7^2 & 4^2 & 5^2 & 4^1 \\ 4^2 & 4^1 & 6^1 & 7^1 & 4^2 \end{pmatrix}. \quad (6)$$

Next, we compile a planning table for the potential method, using the values of matrix (6) as transportation costs. The algorithm of the potential method itself is applied, the results of which are given in Table 2.

Table 2 – Planning table for the potential method of the transport problem for example 2

Planning matrix	***Consumers***					***Stocks***
Suppliers	B_1	B_2	B_3	B_4	B_5	
A_1	5^1	6^1	4^1 100	5^2 100	6^2	200
A_2	6^2	3^1 100	4^1	3^2 150	4^2	250
A_3	3^2 150	7^2	4^2	5^2	4^1	150
A_4	4^2 50	4^1 100	6^1	7^1	4^2 150	300
Needs	200	200	100	250	150	900

Thus, the optimal plan for the task was:

$$X = \left\{ \begin{matrix} x_{13}^1 = 100\,, x_{14}^2 = 100, x_{22}^1 = 100\,, x_{24}^2 = 150, x_{31}^2 = 150\,, \\ x_{41}^2 = 50, x_{42}^1 = 100\,, x_{45}^2 = 150 \end{matrix} \right\}.$$

The value of the objective function is:

$$F = 100 \cdot 4 + 100 \cdot 5 + 100 \cdot 3 + 150 \cdot 3 + 150 \cdot 3 + \\ +50 \cdot 4 + 100 \cdot 4 + 150 \cdot 4 = 3300.$$

The optimal plan record shows not only the value of the transportation itself, but also the type of transport that should be used to transport this cargo.

3. Now let's consider another option for using two types of transport. Let's assume that for each "supplier-consumer" pair, percentage ratios of the amount of cargo that can be transported by one or another mode of transport are given in advance.

These can be probabilistic indicators that can be determined, for example, by the number of transport units of a particular mode of transport that are actually available for transportation at the moment.

Such relationships can be given in the form of a probability matrix *P*, in which the values for the first type of vehicle will be indicated. The corresponding values for the second type can be obtained by subtracting from unity.

$$P = \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{12} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ p_{m1} & p_{m2} & \cdots & p_{mn} \end{pmatrix}, \quad (7)$$

where $p_{ij} \in [0;1]$.

It should be noted that in the case when the coefficients p_{ij} are obtained on the basis of statistical data and can have a probabilistic interpretation, then within the framework of this model they are considered as deterministic and predetermined parameters. In the model, cargo transportation by different modes of transport is not sequential. For each “supplier–consumer” pair, the cargo is distributed between the modes of transport in fixed shares, determined by the matrix *P*.

It should be noted that there are also possible cases when p_{ij}=0 and p_{ij}=1, that is, when cargo in some individual connections is delivered by a vehicle of only the second or only the first type, respectively.

Then the calculation of transportation values will be determined by the expressions:

$$\begin{cases} x_{ij}^1 = p_{ij} \cdot x_{ij} \\ x_{ij}^2 = \left(1 - p_{ij}\right) \cdot x_{ij} \\ x_{ij} = x_{ij}^1 + x_{ij}^2 \end{cases}. \quad (8)$$

Then, taking into account (8), the objective function of the specified problem model will have the following form:

$$F = \left(C_{11}^1 \cdot p_{11} + C_{11}^2\left(1 - p_{ij}\right)\right) \cdot x_{11} + \cdots + \left(C_{mn}^1 \cdot p_{mn} + C_{mn}^2(1 - p_{mn})\right) \cdot x_{mn} = \\ = \sum_{i=1}^{m} \sum_{j=1}^{n} \left(C_{ij}^1 \cdot p_{ij} + C_{ij}^2\left(1 - p_{ij}\right)\right) \cdot x_{ij} \to min\,.$$

In this case, to reduce such a problem to the classical interpretation, it is necessary to create a two-dimensional matrix of transportation costs, each element of which is a minimum with a linear expression of the corresponding elements from the initial three-dimensional matrix of transportation costs (1), which are located in the same line relative to the coordinate axis of the mode of transport.

The new cost matrix will look like this:

$$\begin{pmatrix} C_{11}^1 \cdot p_{11} + C_{11}^2(1 - p_{11}) & C_{12}^1 \cdot p_{12} + C_{12}^2(1 - p_{12}) & \cdots & C_{1n}^1 \cdot p_{1n} + C_{1n}^2(1 - p_{1n}) \\ C_{21}^1 \cdot p_{21} + C_{21}^2(1 - p_{21}) & C_{22}^1 \cdot p_{22} + C_{22}^2(1 - p_{22}) & \cdots & C_{2n}^1 \cdot p_{2n} + C_{2n}^2(1 - p_{12}) \\ \cdots & \cdots & \cdots & \cdots \\ C_{m1}^1 \cdot p_{m1} + C_{m1}^2(1 - p_{m1}) & C_{m2}^1 \cdot p_{m2} + C_{m2}^2(1 - p_{m2}) & \cdots & C_{mn}^1 \cdot p_{mn} + C_{mn}^2(1 - p_{mn}) \end{pmatrix}. \quad (9)$$

Next, we similarly apply the potential method to solve the transport problem.

Consider numerical example 3.

As input values of the transport problem, we take the values of the previous example 1. Also, as a condition of the model of such a problem, the probability matrix of the distribution of cargo by modes of transport between each supplier and each consumer is given:

$$P = \begin{pmatrix} 0{,}52 & 0{,}4 & 0{,}64 & 0{,}88 & 0{,}19 \\ 0{,}35 & 1 & 0{,}81 & 0{,}63 & 0{,}48 \\ 0{,}2 & 0{,}65 & 0{,}34 & 0 & 0{,}78 \\ 0{,}45 & 0{,}7 & 0{,}28 & 0{,}15 & 0{,}59 \end{pmatrix}. \quad (10)$$

To solve such a problem, we compose a final cost matrix according to expression (9):

$$\begin{pmatrix} 6{,}44 & 7{,}8 & 4 & 6{,}76 & 6{,}38 \\ 6{,}35 & 3 & 4{,}76 & 6{,}78 & 4{,}48 \\ 3{,}4 & 7{,}65 & 5{,}02 & 5 & 4{,}22 \\ 4{,}9 & 5{,}5 & 7{,}44 & 7 & 6{,}95 \end{pmatrix}. \quad (11)$$

Now we compile a planning table for the potential method, using the values of matrix (11) as transportation costs.

The algorithm of the potential method itself will be applied, the results of which are given in Table 3.

Table 3 – Planning table for the potential method of the transport problem for example 3

Planning matrix	***Consumers***					***Stocks***
Suppliers	***B_1***	***B_2***	***B_3***	***B_4***	***B_5***	
A_1	6,44	7,8	4 100	6,76 100	6,38	200
A_2	6,35	3 200	4,76	6,78	4,48	250
A_3	3,4	7,65	5,02	5 50	4,22 100	150
A_4	4,9 200	5,5	7,44	7 100	6,95	300
Needs	200	200	100	250	150	900

Thus, the optimal plan for the task was:

$$X = \begin{Bmatrix} x_{13}^1 = 100\,, x_{14}^2 = 100, x_{22}^1 = 100\,, x_{24}^2 = 150, x_{31}^2 = 150\,, \\ x_{41}^2 = 50, x_{42}^1 = 100\,, x_{45}^2 = 150 \end{Bmatrix}.$$

The value of the objective function is:

$$F = 100 \cdot 4 + 100 \cdot 6{,}76 + 200 \cdot 3 + 50 \cdot 5 + 100 \cdot 4{,}22 + \\ +200 \cdot 4{,}9 + 100 \cdot 7 = 4252\,.$$

Conclusions. The paper considers the formulation and constructs three generalized models of the transport problem in the case of cargo delivery by two different types of vehicles. It is shown that the use of the classical transport formulation is possible only after the appropriate transformation of the three-dimensional matrix of transportation costs into a two-dimensional one. Depending on the transportation conditions, three approaches are proposed:

1. Total cost model, when cargo is sequentially transported by both modes of transport.
2. A model for choosing the optimal mode of transport that minimizes costs on each connection.
3. Model of probabilistic or partial distribution of cargo between modes of transport.

For each model, procedures for reducing to a classical transport problem are proposed and the application of the potential method to find the optimal transportation plan is demonstrated. The numerical examples provided confirm the operability of the developed models and the versatility of the proposed approach in planning freight transportation in multi-transport schemes.

The proposed models allow taking into account various logistical conditions, such as restrictions on the share of transportation by certain transport, different modes of vehicle availability, and also provide the possibility of flexible transportation planning based on the criterion of cost minimization.

Prospects for further research.

1. Generalization of the model to the case of more than two modes of transport. Further research may be aimed at developing models in which the number of available modes of transport exceeds two. This requires the formation of multidimensional cost matrices, the expansion of approaches to their reduction to a two-dimensional form, and the creation of universal optimization algorithms. Of particular note is the construction of procedures that allow automatically selecting a combination of vehicles for each direction of transportation and ensuring cost minimization in multimodal logistics chains.

2. Taking into account the technical and operational characteristics of vehicles. A promising direction is to include in the model restrictions on carrying capacity, dimensions, fuel consumption, environmental standards, availability of transport units and other technological parameters. Such an expansion will allow adapting the mathematical model to real transportation conditions. In addition, it is advisable to investigate the mechanisms of integrating these restrictions into the classical transport formulation or to form a new modified model that will combine cost and technical criteria in a single optimization approach.

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Славич В. П., Білоущенко Д. Ю. МОДЕЛЬ ТРАНСПОРТНОЇ ЗАДАЧІ У ВИПАДКУ ДОСТАВКИ ВАНТАЖУ ДВОМА РІЗНИМИ ВИДАМИ ТРАНСПОРТНИХ ЗАСОБІВ

У статті розглянуто особливості моделювання транспортної задачі у випадку, коли вантаж може доставлятися двома різними видами транспортних засобів. Класична транспортна модель не враховує варіативність вартостей перевезень за типами транспорту, що потребує побудови спеціальних математичних моделей та методів їх зведення до стандартної форми. Метою роботи є побудова спеціальних видів моделі транспортної задачі у випадку доставки вантажу кількістю видів транспортних засобів, більше за два, та запропонування алгоритмів розв'язання таких транспортних задач шляхом зведення за допомогою відповідних процедур до класичної моделі Розроблено три підходи до формалізації багатовидових перевезень. Перший підхід грунтується на послідовному перевезенні вантажу обома видами транспорту, що передбачає формування підсумкової матриці вартостей як суми відповідних елементів тривимірної матриці. Другий підхід передбачає вибір того виду транспорту, який забезпечує найменшу вартість доставки для кожного окремого сполучення «постачальник–споживач». Третій підхід спирається на введення матриці ймовірностей або часткових часток вантажу, які визначають, яка частина вантажу повинна перевозитися кожним видом транспорту. Для кожної з моделей запропоновано процедури зведення тривимірної структури вартостей у двовимірну форму, що дозволяє застосовувати метод потенціалів для пошуку оптимального плану перевезень. Наведені числові приклади підтверджують правильність та практичну придатність запропонованих моделей. Отримані результати можуть використовуватися для оптимізації логістичних схем, управління мультимодальними перевезеннями, а також для розроблення програмних засобів підтримки прийняття рішень у транспортних системах.

Ключові слова: *транспортна задача; взаємодія видів транспорту; метод потенціалів; мультимодальні перевезення; вантажні перевезення; оптимальний план доставки.*

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