

## SUBSTANTIVE MODEL OF SHIP TURN WITH ACCOUNT OF THE LATEST ACHIEVEMENTS IN THEORY AND PRACTICE

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*Today, in the scientific and technical literature on the pivot point, the 2-point scheme of ship turn "Center of Gravity – Pivot Point" is used. According to this scheme, the position of the Pivot Point is calculated from the Center of Gravity, which is not entirely correct, since in fact the ship rotates not around the Center of Gravity, but around the Center of Rotation. The Center of Rotation, in turn, can shift relative to the Center of Gravity, in the presence of longitudinal motion. Failure to take these things into account leads to errors in calculating the position of the Pivot Point and the trajectory of the ship's movement around the Pivot Point. For a long time, the concepts of the Center of Rotation and the Pivot Point were confused. Some researchers believed that the Pivot Point shifts in the direction of the ship's movement, others believed that the Pivot Point is located on the opposite side of the midship, relative to the point of application of the lateral force. The 2-point rotation scheme could not combine these two views. In previous works, the authors of this article proposed to use a 3-point rotation scheme, where the third point is the Center of Rotation. The use of a 3-point turn scheme made it possible to explain the dependence of the position of the Pivot Point on both the longitudinal speed and the point of application of the lateral force. The article develops a new substantive model of the ship's turn and a "Memo for the Ship Handlers". The new substantive model and "Memo..." will reduce control errors and increase maneuvering safety, which is especially important in compressed waters. The results obtained are explained by the fact that, unlike known approaches: the new substantive model is built on the basis of a 3-point turn scheme, which takes into account an additional point - the Center of Rotation; the position of Pivot Point is determined relative to the Center of Rotation, and not relative to the Center of Gravity; the position of the Pivot Point is determined on the plane, through the abscissa and ordinate of the Pivot Point.*

**Key words:** navigation safety; 3-point ship's turn scheme; Pivot Point; Center of Rotation; maneuvering in restricted waters.

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**Introduction.** If an observer is on board a vessel that is making a turn, it seems to him that the vessel is rotating around some point, which is called the pole of rotation (PP). In fact, the vessel is currently making two movements: lateral and rotational around the center of rotation (CO). It is the sum of these two real movements that gives the false impression of rotation around another point – the Pivot Point. At the same time, an alternative vision of the rotation process has led to the emergence and development of the concept of the Pivot Point, which makes it possible to optimize the processes of vessel rotation, which is especially important in compressed waters. Where is the Pivot Point, what does its position depend on, how to change it are very important questions for the ship handler?

Today, in the scientific and technical literature on the Pivot Point, a 2-point scheme of vessel rotation "Center of Gravity – Pivot Point" is used. According to this scheme, the position of the Pivot Point is calculated from the Center of Gravity (CG), which is not entirely correct, since in fact the vessel rotates not around the Center of Gravity, but around the Center of Rotation. The Center of Rotation, in turn, can shift relative to the Center of Gravity, in the presence of longitudinal motion. Failure to take these things into account leads to errors in calculating the position of the PP and the trajectory of the vessel's movement around the PP. For a long time, the concepts of the Center of Rotation and the Pivot Point were confused. Some researchers believed that the Pivot Point shifts in the direction of the vessel's movement, others believed that the Pivot Point is located on the opposite side of the middle, relative to the point of application of the lateral force. The 2-point rotation scheme could not combine these two views. In work [1], the authors of the article proposed to use a 3-point rotation scheme, where the third point is the Center of Rotation. The use of a 3-point rotation scheme made it possible to explain the dependence of the PP position on both the longitudinal speed and the point of application of the lateral force. In the general case, in the presence of longitudinal speed and applied lateral force, the Pivot Point is located in the plane of the

vessel's movement and is determined in the vessel-related coordinate system by the abscissa and ordinate of the pole of rotation [7] and coincides with the Center of Turning Circle (CTC). In this work, based on the 3-point scheme of rotation, taking into account the expanded idea of the position of the PP in the plane of the vessel's movement, the main variants of the vessel's movement speeds and applied controls are considered, the corresponding positions of the Center of Rotation, abscissas and ordinates of the PP are determined. This information is necessary for ship handlers to make correct management decisions.

**Problem statement.** To develop a meaningful model of ship's rotation and «Memo for the Ship Handlers» based on the 3-point scheme of ship's rotation.

**Analysis of recent research and publications.** The study of the behavior of the Center of Rotation and the Pivot Point has previously been considered by many authors.

Thus, in work [2] the author studies the behavior of the Center of Rotation of a vessel (the author calls it the Pivot Point) using the example of two tugboats that push the vessel sideward. When longitudinal speed appears, the vessel begins to rotate. The author explains this effect by changing the force arms of the tugboats due to the displacement of the Center of Rotation in the direction of the vessel's movement. Also?

In the book [3], for a simplified vessel model, the authors obtained analytical dependences of the position of the Pivot Point, as an apparent point of rotation of the vessel, on the shoulder of application of the lateral force and constructed graphs. In the obtained results, the shoulder of application of the lateral force and the corresponding position of the Pivot Point were counted from the midship of the frame, and not from the Center of Rotation

Experiments with the Center of Rotation (the author also calls it the pole of rotation) were also carried out in the port of Revel [4]. According to the authors of the Porto-Revel Shiphandling Course manual – when the ship is stopped in the water, PP coincides with the Centre of Gravity of the ship, – when the ship is gaining speed, PP moves along ships in the same direction as the movement.

In [5], the author showed how the movement of the Pivot Point can be used to improve the maneuverability of a large sailing vessel.

The properties of the Pivot Point, as apparent point, were considered in work [6]. The author proposed a formula for determining the position of the Pivot Point through the lateral velocity of the Center of Gravity and the angular velocity of rotation of the vessel around the Center of Gravity. The author also linearized the mathematical model of the channels of lateral and angular motions and obtained the values of lateral and angular velocity for the steady motion of the vessel.

In work [7], using the example of a vessel departing from the berth on sternway, the author showed that the use of existing recommendations for vessel control at that time leads to the vessel hitting the berth. It was believed that the vessel rotates around the Center of Rotation, which when moving in reverse is shifted back. In fact, the vessel rotated around the Pivot Point, which, on the contrary, was shifted forward, which led to the vessel hitting the berth. The author also gave considerations regarding the Center of Rotation, which should be located between the Center of Gravity and the Center of lateral hydrodynamic resistance (COLR), which is shifted in the direction of vessel movement. The maximum COLR displacement, according to the author, is up to 10% of the ship's length. The author also emphasized that the Center of Rotation and the Pivot Point are two different centers.

In work [8], for the first time, the condition for determining the position of the pole of rotation is written not in the scalar form  $V_y + \omega_z R = 0$ , which was used by predecessors, but in the vector form  $\mathbf{V} + \boldsymbol{\omega} \times \mathbf{R} = \mathbf{0}$ . The use of the vector equation showed that the scalar form is a special case of the vector form, which determines only one of the three components of the vector  $\mathbf{R}$  – the abscissa of the Pivot Point  $R_x$ . The use of the vector form also allows us to determine the ordinate  $R_y$  and applicate  $R_z$  of the Pivot Point, that is, in the general case, the Pivot Point is located in the three-dimensional space of the coupled coordinate system (CCS), and not only on its longitudinal

axis  $OX_1$ . For practical maneuvering, only the abscissa and ordinate of the Pivot Point are used, which together determine the position of the center of circulation.

Capt. P. Butusina in his work [9] considered the process of turning the vessel and its interaction with the hydrodynamic forces that arise during this process.

In his works [10], [11], [12], the author identifies three special centers of rotation: the Center of Turning Circle (the center of planetary rotation  $E$  fixed on the Earth's surface), the Center of Rotation of the vessel  $S$  and the Pivot Point (the apparent center of rotation  $P$ ). The author also draws attention to the new view of the Pivot Point (in the author's words, the new concept of the Pivot Point), which differs from the traditional one in that the Pivot Point is a apparent center of rotation, its position does not depend on the longitudinal speed of the vessel and it is not the Center of Rotation of the vessel. The work also provides examples of using the new concept of the Pivot Point for practical maneuvering. In addition, in article [10] the author calls for teaching future shiphandlers and retraining all working shiphandlers in the correct methods of ship control according to the concept of the Pivot Point.

In paper [13], the author reflects on the process of ship turning and generally supports the provisions proposed by Dr. Seong-Gi Seo in his works. In paper [14], he summarizes the views of authors who study ship turning problems and comes to the conclusion that shiphandlers' training should be carried out based on the new theory.

Fundamental research on ship control is given in the book [15], in particular, in section 7.2.4. Pivot Point shows the current state of ideas about the pivot point and its use for ship control.

In the article [16], a refined scheme for calculating the position of the Center of Rotation and the Pivot Point was developed, a control scheme with a bow and stern thruster was investigated, control lines were found on which the specified ship movement is implemented (around the Pivot Point, without lateral speed, without angular speed), and the steering distribution coefficient was investigated.

In study [17], a refined method is presented for a single-screw vessel, providing formulas and graphs to locate the pivot point relative to a fixed reference point (center of gravity or middle frame) based on the enhanced scheme. These results enable practical application for both automated and manual vessel control systems.

The study [18] focuses on processes for automatically controlling vessel rotation around the pivot point with zero drift. Two linearized control models are examined: one for a single-screw conventional vessel without a bow thruster, and another for the same vessel equipped with a bow thruster. For each control scheme under steady-state conditions, control strategies were derived to enable circulation around the pivot point without a drift angle. These strategies enhance safety and efficiency by narrowing traffic lanes, reducing hydrodynamic resistance and fuel consumption, and facilitating technological operations like mooring. The effectiveness of these methods was validated through mathematical modeling of the automatic mooring process of the MSC Container Ship without drift angle. This simulation was conducted on a custom-developed imitation modeling stand based on the Navi Trainer 5000 navigation simulator.

In [19], the behavior of the rotation center and the Pivot Point was investigated, and the following results were obtained: it has been demonstrated that the vessel's rotation center, pivot point, and gravity center are distinct points that generally do not coincide; the rotation center represents the point around which the vessel experiences the greatest angular acceleration from an applied torque. This point shifts gradually along a hyperbolic trajectory relative to the gravity center, depending on the vessel's speed; the existence of a pivot point is both necessary and sufficient when the linear lateral velocity and angular velocity relative to the rotation center are present; the Pivot Point is highly dynamic and can shift rapidly when the angular velocity fluctuates near zero; control lines are defined that provide straight-line motion, drift-free motion, and motion around the Pivot Point; optimal control strategies for vessel motion around the pivot point were developed; the accuracy of the proposed methods and algorithms was validated through mathematical modeling using the Navi Trainer 5000 navigation simulator.

In the videos [20–23], the author Dr. Knud Benedikt shows and explains his version of the Pivot Point. He shows how the use of various sources of lateral forces (rudder blade, stern and bow thrusters, wind) affects the position of the Pivot Point and concludes that the position of the Pivot Point depends on the ratio of angular and lateral velocities. He also shows that the position of the Pivot Point can be determined from the tangential velocities of the bow and stern. In addition, he shows how the position of the Pivot Point changes when performing a Turning Circle, in shallow water and how the position of the Pivot Point changes relative through the water and relative to the ground during maneuvering.

In the article [24], it is shown how to use the position of the Pivot Point when mooring with two tugs.

The video [25] shows a complex approach of a vessel and mooring in a narrow place using knowledge of the vessel's Pivot Point, the action of rudder forces, and other lateral forces.

The author of the article [26] believes that the closer the Pivot Point is to the middle, the greater the turning radius of the vessel. In terms of hull shape, a fuller-shaped vessel, such as a bulk carrier or tanker, has a Pivot Point closer to the bow than vessels with cruising contours.

**Purpose and objectives of the study.** The purpose of the study is to reduce ship control errors in compressed waters and increase the safety of navigation in general.

The goal is achieved by using a new meaningful model and “Sailor’s Notes”, developed on the basis of a 3-point turn scheme, taking into account the latest views on the theory of ship turning.

The objectives of the study are to develop a new, meaningful model of ship turning and «Memo for the Ship Handlers».

**The main part.** To consider the issue of turning a ship, we will first take a single-screw conventional ship without additional devices. That is, the ship has an engine connected to a propeller shaft that passes through the deadwood and ends with a propeller. Behind the propeller is a rudder. This is the basic model from which we will begin to consider the issue of turning a ship.

As noted above in the introduction, the existing 2-point ship turning scheme cannot explain the dependence of the position of the Pivot Point on the speed of the ship and simultaneously on the point of application of the steering force. The authors of the article believe that the reason for this contradiction is the absence of an additional point between the Center of Gravity and the Pivot Point. Such a point is the Center of Rotation. The theoretical justification for the need to take into account the Center of Rotation in the ship turning scheme is provided in the article [1]. The Center of Rotation moves in the direction of the vessel's movement, and the Pivot Point is counted from the Center of Rotation, which explains the dependence of the position of the PP on the longitudinal speed of the vessel. At the same time, the position of the PP also depends on the point of application of the lateral force relative to the Center of Rotation. If the lateral force is applied in the stern of the vessel, then the PP will be in the bow and vice versa. The use of a 3-point rotation scheme solves the problem of inconsistency of the 2-point scheme, namely, it allows explaining the dependence of the position of the PP on both the longitudinal speed and the point of application of the lateral force.

Fig. 1 shows a 3-point scheme of the vessel's rotation, which is the basis of the new substantive model. The diagram shows:

– the first base point – Center of Gravity (CoG). The position of the Center of Gravity can always be taken from the current ship load plan at the moment;

– the second base point – Center of Rotation (CoR). The Center of Rotation is located at the point relative to which an arbitrary torque has the greatest efficiency (creates the greatest angular acceleration). It can also be said that the position of the Center of Rotation is determined by a compromise between increasing the moment of inertia of rotation and decreasing the moment of hydrodynamic resistance (the Center of Rotation is located between the Center of Gravity and the Center of Lateral Resistance (CLR). The Center of Rotation can also be called the true center of rotation. In the absence of longitudinal speed, the position of the Center of Rotation coincides with the Center of Gravity. With the appearance of longitudinal speed, the Center of Rotation begins to shift. The displacement of the Center of Rotation relative to the Center of Gravity occurs slowly, within the hull of the vessel, depends on the longitudinal speed of the vessel.

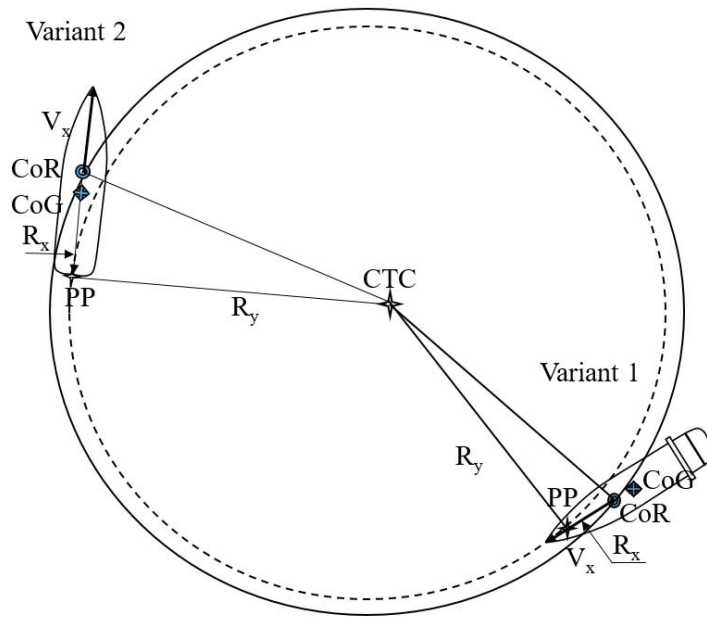


Figure 1 – 3-point ship turning diagram

The Center of Rotation shifts towards the oncoming flow, and is determined by the formula:

$$\Delta x = \frac{L}{2} \left( 1 - \frac{V_{\max}}{\eta V_x + V_{\max}} \right), \tag{1}$$

where  $L$  is the length of the vessel,  $V_{\max}$  is the maximum speed of the vessel,  $V_x$  is the longitudinal speed of the vessel,  $\eta$  is the coefficient determined by the maximum displacement of the center of rotation. The estimate of the maximum displacement of the center of rotation (up to 10% of the vessel length) is given in [7]. In the experimental studies of the authors [16], conducted on the Navi Trainer 5000 simulator, it was established that the magnitude of the displacement of the center of rotation lies within (10–20 %) of the vessel length. The shoulders and moments of all lateral forces acting on the vessel are calculated relative to the center of rotation.

– the third base point is the Pivot Point. The PP is the point around which the vessel turns. The position of the Pivot Point relative to the center of rotation is determined by the abscissa  $R_x$  and ordinate  $R_y$  of the Pivot Point

$$\begin{cases} R_x = -\frac{V_y}{\omega_z}, \\ R_y = \frac{V_x}{\omega_z}, \end{cases} \tag{2}$$

where  $V_y$  is the lateral velocity of the center of rotation,  $\omega_z$  is the angular velocity of rotation in the yaw channel,  $V_x$  is the longitudinal velocity of the vessel.

From system (2) it is clear that the PP can move very quickly from to, if the angular velocity fluctuates around zero.

$$R_x = -\frac{V_y}{-0} = +\infty, R_x = -\frac{V_y}{+0} = -\infty, R_y = \frac{V_x}{-0} = -\infty, R_y = \frac{V_x}{+0} = +\infty$$

The graph of the dependence of the PP abscissa on the shoulder of application of the lateral force is a hyperbola [2], Fig. 2.

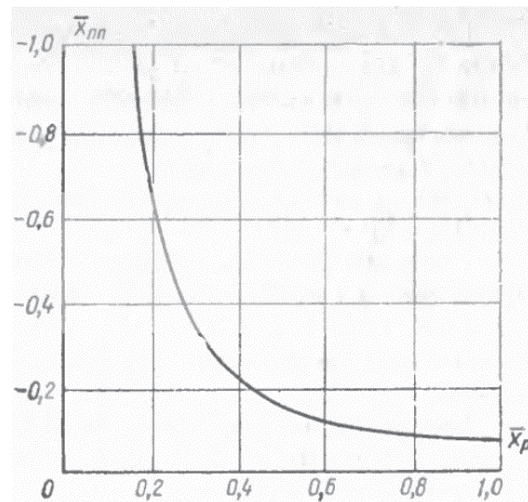


Figure 2 – Graph of the dependence of the abscissa of the PP on the arm of the lateral force  $x_p$

From the above graph it can be seen that when a lateral force (control or external influence) is applied to the vessel, which has a shoulder  $x_p$  to the CoR, the PP moves to a point  $x_{nn}$ , which is located on the other side of the CoR, as evidenced by the different signs of the shoulders. The further the point of application of the lateral force  $x_p$  from the CoR (the greater the shoulder of the lateral force  $x_p$ ), the closer the PP is to the CoR and vice versa. When the point of application of the lateral force approaches the Center of Rotation (the shoulder decreases), the PP tends to infinity.

From formula (2) the following conclusions follow:

– if  $V_y = 0, \omega_z \neq 0, R_x = \frac{0}{\omega_z} = 0$ , the vessel circulates without drift of the CoR point, the PP is in the CoR;

– if  $V_y \neq 0, \omega_z \neq 0, R_x = -\frac{V_y}{\omega_z}$ , the vessel circulates without drift angle of the CoR point, the greater the drift angle, the further the PP from the CoR;

– if  $V_y \neq 0, \omega_z = 0, R_x = -\frac{V_y}{0} = \pm\infty$ , the ship moves in a straight line with the drift angle of the point CoR, PP is at infinity;

– if  $V_y = 0, \omega_z = 0, R_x = -\frac{0}{0}$ , the ship moves in a straight line without drift. The formula shows uncertainty, but it is physically clear that in this case PP must also be at infinity.

PP can also be called the apparent Center of Rotation.

The abscissa and ordinate of the Pivot Point determine the position of the vessel in Turning Circle, Fig. 1. If the abscissa of the PP  $R_x > 0$ , the position of the vessel in Turning Circle is with the bow inward, option 1. This position is typical for vessels with insufficient control, for which the number of independent controls is less than 3 [1]. Such vessels include conventional single-screw vessels that have two independent controls (telegraph deviation and rudder deviation). If the abscissa of the Pivot Point  $R_x < 0$ , the position of the vessel in Turning Circle is with the bow outward, option 2. This position can be achieved on vessels with excessive control, for which the number of independent controls is greater than or equal to three. Such vessels include vessels equipped with an additional bow and/or stern thruster, vessels with a dynamic positioning system. If the abscissa of the Pivot Point  $R_x = 0$ , the diametrical plane of the vessel is tangent to the Turning

Circle and the vessel moves without drift. This position can also be achieved on vessels with redundant controls;

– the fourth base point is the Center of Turning Circle (CTC). CTC is the center of the trajectory along which the vessel moves. The position of the Center of Turning Circle in the coordinate system associated with the vessel is determined by the radius of Turning Circle drawn from the CTC to the CoR of the vessel. The radius of Turning is determined through the abscissa and ordinate of Pivot Point according to the following formula:

$$R_c = \sqrt{R_x^2 + R_y^2} \tag{3}$$

**Memo for the Ship Handlers on the position of the Center of Rotation and Pivot Point**

Below are 6 basic maneuvering options, for which the position of the Center of Rotation and Pivot Point are determined according to the new meaningful model of turn. For all the options given, the Center of Gravity is assumed to be located amidships (the vessel is on a straight keel, has no trim).

*Option 1.* The vessel has no longitudinal, lateral and angular velocities. There is also no lateral force,  $V_x = 0; V_y = 0; \omega_z = 0; F_y = 0$ . In this case, the Center of Rotation is located amidships ( $\Delta x = 0$ , according to formula (1), the position of the abscissa and ordinate of the

pole of rotation, according to formula (3), is not determined ( $R_x = -\frac{0}{0}, R_y = \frac{0}{0}$ ).



Figure 3 – The ship lies motionless in the drift

*Option 2.* The ship has no longitudinal motion. The bow thruster (BST) creates a lateral force. 2 motions are formed: rotational around the center of gravity and translational, in the direction of application of the lateral force,  $V_x = 0, V_y = 0, \omega_z \neq 0, F_y \neq 0$ . Fig. 4 shows the velocity vectors of lateral and rotational motion along the length of the ship, and Fig. 5 shows the vectors of total velocities. In this case, the center of rotation is located at the midship (according to formula (1)

$\Delta x = 0$ ), and the Pivot Point is behind the midship ( $R_x = -\frac{V_y > 0}{\omega_z > 0} < 0, R_y = \frac{V_x = 0}{\omega_z \neq 0} = 0$ ).

Fig. 4 shows the velocity vectors of lateral and rotational motion along the length of the vessel, the positions of the Center of Gravity and Center of Rotation. Fig. 8 shows the vectors of total velocities, the positions of the Center of Gravity, Center of Rotation, Pivot Point and Center of Turning Circle from the action of the lateral force of the rudder at the bow.

Due to the mismatch between the position of the pole of rotation and the center of gravity, the vessel also receives a slight longitudinal velocity from the action of centrifugal force.

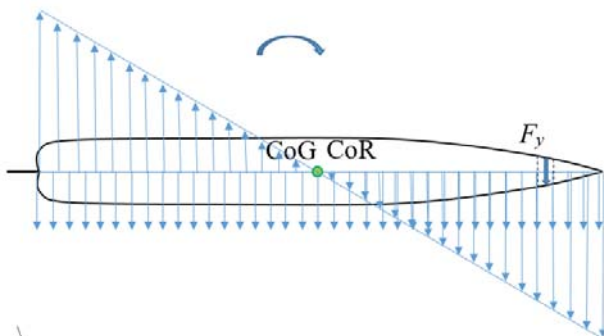


Figure 4 – Velocity vectors of lateral and rotational motion along the length of the vessel

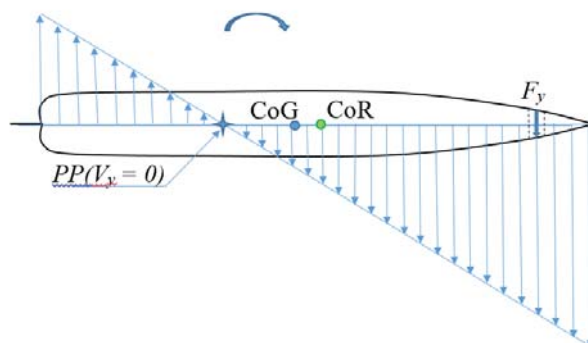


Figure 5 – Vectors of total speeds along the length of the vessel

*Option 3.* The vessel has a longitudinal velocity, lateral velocity and yaw angular velocity are absent, lateral force is absent,  $V_x \neq 0, V_y = 0, \omega_z = 0, F_y = 0$ . In this case, the center of rotation of the vessel is shifted forward (according to formula (1)  $\Delta x > 0$ ), and the position of the Pivot Point is not determined (abscissa of the Pivot Point is  $R_x = -\frac{V_y = 0}{\omega_z = 0}$ , ordinate of the Pivot Point is

$$R_y = \frac{V_x \neq 0}{\omega_z = 0} = \pm\infty).$$

Fig. 6 shows the displacement of the Center of Rotation relative to the Center of Gravity in the direction of vessel movement.

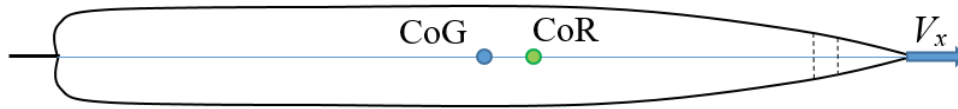


Figure 6 – Displacement of the Center of Rotation relative to the Center of Gravity in the direction of the vessel’s movement

*Option 4.* The vessel has longitudinal motion, lateral motion and rotational motion due to the shifted rudder,  $V_x \neq 0, V_y \neq 0, \omega_z \neq 0, F_y \neq 0$ . In this case, the center of rotation of the vessel is shifted forward (according to formula (1)  $\Delta x > 0$ ), and the position of the Pivot Point is in the plane of the vessel’s movement (the Pivot Point is in front of the Center of Rotation,  $R_x = -\frac{V_y \neq 0}{\omega_z \neq 0} > 0$ , the ordinate of Pivot Point is positive or negative, depending on the sign of  $\omega_z$ ,  $R_y = \frac{V_x \neq 0}{\omega_z \neq 0}$ ). Fig.

7 shows the velocity vectors of lateral and rotational motion along the length of the vessel, the positions of the Center of Gravity and Center of Rotation. Fig. 8 shows the vectors of total velocities, the positions of the Center of Gravity, Center of Rotation, Pivot Point and Center of Turning Circle from the action of the lateral force of the rudder.

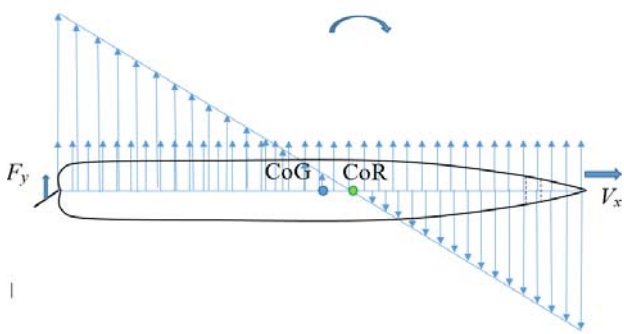


Figure 7 – Velocity vectors of lateral and rotational motion along the length of the vessel from the action of the lateral force of the rudder

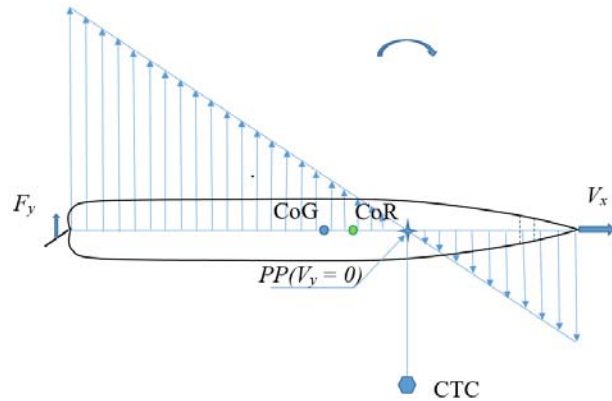


Figure 8 – Vectors of total velocities along the length of the vessel from the action of the lateral force of the rudder

Under the action of the lateral force of the rudder (in the stern), the Pivot Point is formed in the bow and due to this the stern has a much greater displacement to the side than the bow, which must be taken into account when moving along curved sections of the path. Due to the mismatch between the position of the Pivot Point and the Center of Gravity, the vessel gains a small reverse speed from the action of centrifugal force.



*Option 5.* The vessel has a longitudinal velocity, as well as lateral velocity and angular velocity, created by the lateral force of the tug in the bow of the vessel  $V_x \neq 0, V_y \neq 0, \omega_z \neq 0, F_y \neq 0$ .

In this case, the Center of Rotation of the vessel is shifted forward (according to formula (1)  $\Delta x > 0$ ), and the position of the Pivot Point is in the plane of movement of the vessel (the abscissa of the

Pivot Point is behind the Center of Rotation,  $R_x = -\frac{V_y \neq 0}{\omega_z \neq 0} < 0$ , the ordinate of the Pivot Point is

positive or negative, depending on the sign  $\omega_z$  (determined by the direction of the applied force),

$$R_y = \frac{V_x \neq 0}{\omega_z \neq 0}.$$

Fig. 9 shows the vectors of the lateral and rotational motion speeds along the length of the vessel, the positions of the Center of Gravity and Center of Rotation, and Fig. 10 shows the vectors of total velocities, positions of the Center of Gravity and Center of Rotation, Pivot Point position and Center of Turning Circle from the action of the lateral force of the tug at the bow. Due to the mismatch between the position of the pole of rotation and the center of gravity, the vessel gains a small forward speed from the action of centrifugal force.

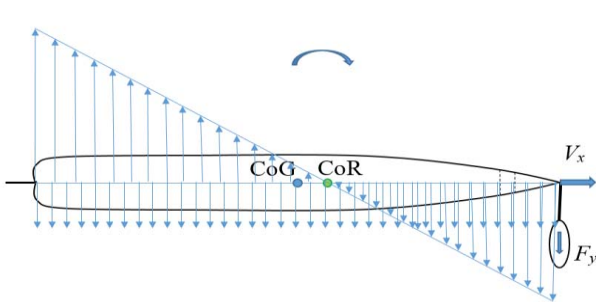


Figure 9 – Velocity vectors of lateral and rotational motion along the length of the vessel due to the action of the lateral force of the tug at the bow

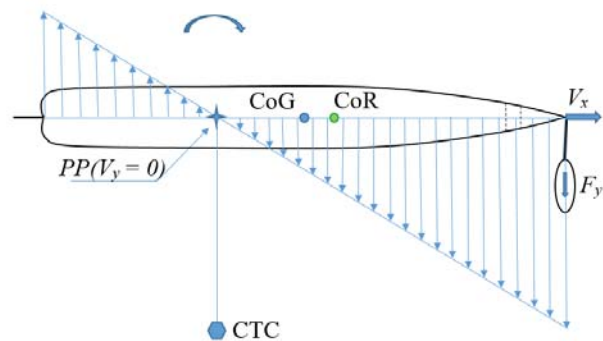


Figure 10 – Vectors of total velocities along the length of the vessel due to the action of the lateral force of the tug at the bow

*Option 6.* The vessel has a longitudinal velocity, as well as lateral velocity and angular velocity, created by the lateral force of the tug in the stern of the vessel,  $V_x \neq 0, V_y \neq 0, \omega_z \neq 0, F_y \neq 0$ . In this case, the center of rotation of the vessel is shifted forward (according to formula (1)  $\Delta x > 0$ ), and the position of the Pivot Point is in the plane of the vessel's

movement (the abscissa of the Pivot Point is in front of the Center of Rotation,  $R_x = -\frac{V_y \neq 0}{\omega_z \neq 0} > 0$ , the ordinate of the Pivot Point is positive or negative, depending on the sign  $\omega_z$  (determined by the

direction of the applied force),  $R_y = \frac{V_x \neq 0}{\omega_z \neq 0}$ ). Fig. 11 shows the velocity vectors of lateral and

rotational motion along the length of the vessel, the positions of the Center of Gravity and Center of Rotation. Fig. 12 shows the vectors of total velocities, positions of the Center of Gravity and Center of Rotation, Pivot Point and Center of Turning Circle from the action of the lateral force of the tug at the stern. Due to the mismatch between the position of the Pivot Point and the Center of Gravity, the vessel gains a small reverse speed from the action of centrifugal force.

When the vessel moves backward, the Center of Rotation also shifts backward. The position of the Pivot Point is counted from the Center of Rotation. The abscissa and ordinate of the Pivot Point are determined relative to the Center of Rotation similarly to the options for applying lateral force considered above.

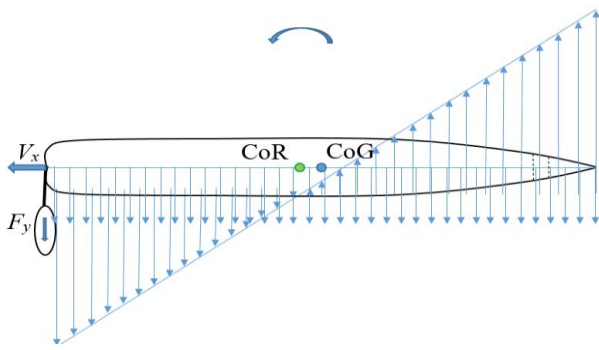


Figure 11 – Velocity vectors of lateral and rotational motion along the length of the vessel due to the action of the lateral force of the tug at the stern

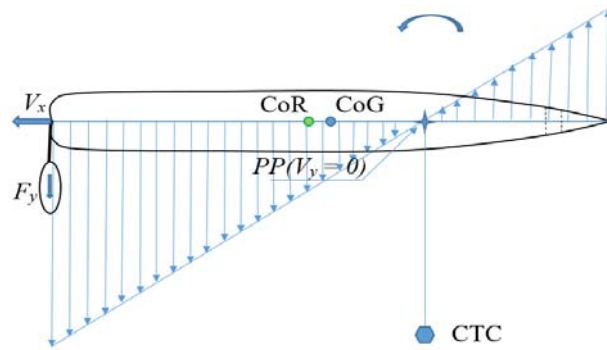


Figure 12 – Vectors of total velocities along the length of the vessel due to the action of the lateral force of the tug at the stern

**Discussion.** The issues of maneuvering a vessel in compressed waters are considered. A search and analysis of literary sources in which these issues were considered were carried out. A new substantive model of vessel turning has been developed, and a “Memo for the Shiphandlers” has been formulated. The use of the new substantive model and “Memo...” will allow reducing errors in vessel control, which is especially important in compressed waters. The results obtained are explained by the fact that, unlike known approaches: the new substantive model uses a 3-point turning scheme, which takes into account an additional point - the Center of Rotation; the position of the Pivot Point is determined relative to the Center of Rotation, and not relative to the Center of Gravity; the position of the Pivot Point is determined on the plane, through the abscissa and ordinate of the Pivot Point. The results obtained can be used for manual control of the vessel. Also, the results obtained can and should be included in the process of training shiphandlers in maritime educational institutions, since the use of the old substantive model based on the 2-point turn scheme, due to the inconsistency of ideas and the real behavior of the vessel in compressed waters, can lead to accidents.

Further research may be related to the development of algorithms for the navigator's actions when controlling a vessel in compressed waters, using the theory of fuzzy logic.

**Conclusions.** A new substantive model of ship turning and a "Memo for the Shiphandlers" have been developed. The new substantive model and "Memo..." will reduce control errors and increase maneuvering safety, especially in compressed waters. The results obtained are explained by the fact that, unlike known approaches: the new substantive model is built on the basis of a 3-point turn scheme, which takes into account an additional point – the Center of Rotation; the position of the Pivot Point is determined relative to the Center of Rotation, and not relative to the Center of Gravity; the position of the Pivot Point is determined on the plane, through the abscissa and ordinate of the Pivot Point.

The theoretical value of the obtained results consists in: developing a meaningful model of the vessel's rotation based on a 3-point scheme, taking into account the displacement of the Center of Rotation relative to the Center of Gravity; determining the position of the Pivot Point relative to the Center of Rotation, and not relative to the Center of Gravity; determining the position of the Pivot Point on the plane, through the abscissa and ordinate of the Pivot Point.

The practical value of the obtained results consists in developing a "Memo for the Shiphandlers" based on a new meaningful model, which will reduce errors in vessel movement control and increase navigation safety in general.

## REFERENCES

1. Zinchenko, S., Tovstokoryi, O., Nosov, P., Popovych, I., Kyrychenko, K. (2022). Pivot point position determination and its use for manoeuvring a vessel, *Ships and offshore structures*, Vol.18, Issue 3, 358-364. <https://doi.org/10.1080/17445302.2022.2052480>.
2. Hooyer, H. H. (1991). *Behavior and Handling of Ships*, Cornell Maritime Press.
3. Domin, S. I. et al. (1991). *Keruvannia sudnom*, M.: Transport, 359 с.
4. Port Revel Shiphandling. Course manual 2006.pdf <https://pdfcoffee.com/qdownload/manual-2006-ship-handling-pdf-free.html>.
5. Chase, G. A. (1999). *Sailing Vessel Handling and Seamanship – The Moving Pivot Point*, *The Northern Mariner/Le Marin Du Nord*, Vol. 9. Issue 3, 53–59. <https://doi.org/10.25071/2561-5467.629>.
6. Tzeng, C. Y. (1998). Analysis of The Pivot Point for a Turning Ship, *Journal of Marine Science and Technology*, Vol. 6, Issue 1, 34–44. <https://doi.org/10.51400/2709-6998.2518>.
7. Cauvier, H. (2008), *The Pivot Point, The PILOT*. The official organ of the United Kingdom Maritime Pilots' Association, Vol. 295. <http://www.pilotmag.co.uk/wp-content/uploads/2008/06/pilotmag-295-final-web.pdf>.
8. Artyszuk, J. (2010), *Pivot point in ship manoeuvring*, *Scientific Journals Maritime University of Szczecin*, Vol. 20, Issue 92, 13–24.
9. Butusina, P. (2011). *The Pivot Point Revisited*, *United Kingdom Maritime Pilots' Association "The Pilot"*, Autumn 2011, № 306. <https://ukmpa.org/wp-content/uploads/2016/06/Pilot-306-web.pdf>.
10. Seo, S. G. (2011). *The Use of Pivot Point in Ship Handling for Safer and More Accurate Ship Manoeuvring*, *Proceedings of IMLA*, Vol. 1, Issue 29, 271–280. [https://www.academia.edu/36456506/The\\_Use\\_of\\_Pivot\\_Point\\_in\\_Ship\\_Handling\\_for\\_Safer\\_and\\_More\\_Accurate\\_Ship\\_Manoeuvring](https://www.academia.edu/36456506/The_Use_of_Pivot_Point_in_Ship_Handling_for_Safer_and_More_Accurate_Ship_Manoeuvring).
11. Seo, S. G., Earl, K. (2015). *A Paradigm Shift in Shiphandling (The Pivot Point)*, *Warsash Maritime Academy*, Southampton Solent University.
12. Seo, S. G. (2016). *Safer and More Efficient Ship Handling with the Pivot Point Concept*, *The International Journal on Marine Navigation and Safety of Sea Transportation*, Vol. 10, Issue 4, 605-612. <https://doi.org/12.12716/1001.10.04.09>.
13. Cummins, T. (2020), *Scientific Fact: The 'traditional' understanding of the ship's pivot point is wrong! A review of the ship's pivot point: Science, Maths and Observation*, *Harbour Pilot, Portsmouth International Port*. <https://www.marine-pilots.com/articles/81904-scientific-fact-traditional-understanding-of-ships-pivot-point-is-wrong>.
14. Cummins, T. (2020). *Where is the centre of a ship's rotation?, A review of the ship's pivot point: Science, Maths and Observation*, *Harbour Pilot, Portsmouth International Port*. <https://www.marine-pilots.com/articles/84506-review-of-ships-pivot-point-science-maths-and-observation-where-is-centre-of-ships-rotation>.
15. Fossen, T. I. (2021). *Handbook of marine craft hydrodynamics and motion control: second edition*, *Norwegian university of science and technology*, Wiley.
16. Tovstokoryi, O. M., Zinchenko, S. M., Nahrybelnyi, Ya. A., Tymofeiev, K. V. (2023), *Rozrakhunok velychyny zmishchennia tsentru obertannia sudna*, *Naukovyi visnyk KhSMA*, № 26–27, 44–55. <https://doi.org/10.33815/2313-4763.2023.1-2.26-27.044-055>.
17. Zinchenko, S., Kobets, V., Tovstokoryi O., Kyrychenko, K., Nosov, P., Popovych, I. (2023). *Control of the Pivot Point Position of a Conventional Single-Screw Vessel*, *CEUR-WS.org*, Vol. 3513, p.130–140, *ICST-2023*. <https://ceur-ws.org/Vol-3513/paper11.pdf>.
18. Zinchenko, S., Tovstokoryi, O., Sapronov, O., Petrovskiy, A., Ivanov, A., Tymofeiev, K. (2022). *Development of automatic control methods of vessel rotation around the pivot point without drift*, *Technology Audit and Production Reserves*, Vol. 6(2(68)), 16–21. <https://doi.org/10.15587/2706-5448.2022.269364>.

19. Zinchenko, S., Tovstokoryi, O. (2020). What is the Pivot Point and how to use it to control the vessel, Materials of the XII International Scientific and Practical Conference "Advanced Information and Innovative Technologies for Transport (MINTT – 2020), May 27–29, 2020, Kherson.
20. Dr. Knud Benedict. (2020). Pivot Point Demo. Hochschule Wismar, Institute ISSIMS, Department of Marine Studies. <https://www.youtube.com/watch?v=zc9C5EWIFOE>.
21. Dr. Knud Benedict. (2020). Pivot Point Position at Stopped ship: only thusters, no propulsion, Hochschule Wismar, Institute ISSIMS, Department of Marine Studies. <https://www.youtube.com/watch?v=EYTF9INUET8>.
22. Dr. Knud Benedict. (2020). Pivot Point position for ship moving ahead or astern, Hochschule Wismar, Institute ISSIMS, Department of Marine Studies. <https://www.youtube.com/watch?v=v1I8c6aaFgo&t=16s>.
23. Dr. Knud Benedict. (2020). Pivot Point on specific aspects, Hochschule Wismar, Institute ISSIMS, Department of Marine Studies. [https://www.youtube.com/watch?v=Dqf5sr\\_nNxM&t=9s](https://www.youtube.com/watch?v=Dqf5sr_nNxM&t=9s).
24. Gripoli, M. (2022). Ships & Tugs: Pivot Point. <https://www.standbyengine.it/lessons/ships-tugs-pivot-point>.
25. Brittain, S. (2023). Skillful captain docking a big ship in a narrow place: Pivot point, rudder force, transverse effect. <https://www.youtube.com/watch?v=jP5mdg5pm7s>.
26. Ghosh, S. (2024). What Is The Pivot Point of a Vessel?, Marine Navigation. [https://www.marineinsight.com/marine-navigation/pivot-point-of-a-vessel/#google\\_vignette](https://www.marineinsight.com/marine-navigation/pivot-point-of-a-vessel/#google_vignette).

**Товстокорій О. М. ЗМІСТОВНА МОДЕЛЬ ПОВОРОТУ СУДНА ІЗ ВРАХУВАННЯМ ОСТАННІХ ДОСЯГНЕНЬ ТЕОРІЇ ТА ПРАКТИКИ**

Сьогодні у науково – технічній літературі по полюсу повороту використовується 2-х точкова схема повороту судна «Центр ваги – Полюс повороту». Згідно з цією схемою, положення полюса повороту відраховується від центру ваги, що не зовсім вірно, оскільки фактично судно обертається не навколо центру ваги, а навколо центру обертання. Центр обертання, у свою чергу, може зміщуватися відносно центру ваги, за наявності поздовжнього руху. Не врахування цих речей призводить до похибок розрахунку положення полюса повороту і траєкторії руху судна навколо полюса повороту. Тривалий час поняття центр обертання і полюс повороту плутали. Одні дослідники вважали, що полюс повороту зміщується в сторону руху судна, інші вважали, що полюс повороту розміщується з протилежного боку від міделю, відносно точки прикладання бокової сили. 2-х точкова схема повороту не могла сумістити ці два погляди. У попередніх роботах авторами даної статті запропоновано використовувати 3-х точкову схему повороту, де третьою точкою є центр обертання. Використання 3-х точкової схеми повороту дозволило пояснити залежність положення полюса повороту як від поздовжньої швидкості, так і від точки прикладання бокової сили. У статті розроблено нову змістовну модель повороту судна та «Пам'ятку судноводію». Нова змістовна модель та «Пам'ятка...» дозволять зменшити похибки керування та підвищити безпеку маневрування, що особливо важливо у стиснених водах. Отримані результати пояснюються тим що, на відміну від відомих підходів: нова змістовна модель побудована на основі 3-х точкової схеми повороту, яка враховує додаткову точку – центр обертання; положення полюса повороту визначається відносно центру обертання, а не відносно центру ваги; положення полюса повороту визначається на площині, через абсцису й ординату полюса повороту.

**Ключові слова:** навігаційна безпека; 3-х точкова схема повороту судна; полюс повороту; центр обертання; маневрування в стиснених водах.

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